Econ 101 Problem Set 6

The mythical kingdom of Philhill is ruled by a philosopher-king who donates his time as mediator of all domestic disputes. Since there are no external enemies, there is no need for government spending or taxes. There are also no economic transactions with other countries. The result is that the macroeconomic environment in Philhill can be shown in terms of only consumption (C) and investment (I) spending, as in the table below.

1	2	3	4	5	6	7	8	9	10	11
At a GDP of:	Disposable Income	C1	I1	Y2=C1+I1	I2	Y3=C1+I2	I3	Y4=C1+I3	I4	Y5=C1+I4
0	0	100	200	**300**	100	**200**	50	**150**	150	**250**
500	500	500	200	**700**	100	**600**	100	**600**	200	**700**
1,000	1,000	900	200	**1100**	100	**1000**	150	**1050**	250	**1150**
1,500	1,500	1,300	200	**1500**	100	**1400**	200	**1500**	300	**1600**
2,000	2,000	1,700	200	**1900**	100	**1800**	250	**1950**	350	**2050**
2,500	2,500	2,100	200	**2300**	100	**2200**	300	**2400**	400	**2500**

Part A:

1. Note the relationship between GDP and disposable income in the data of columns 1 and 2. What (admittedly unrealistic) situation is being assumed?

There is no government involved. Hence, there are no taxes and disposable income equals GDP.

2. Note the relationship between consumption and disposable income shown in the data of columns 2 and 3. What kind of cause-and-effect is being described? Calculate the nation's marginal propensity to consume (MPC) from the data given.

There is a positive dependence of consumption on disposable income: consumption increases as Y_d increases. We can also note that this relationship is linear, i.e.,

$C = a + b*Y_d$.

Therefore, $MPC = b = \Delta C/\Delta Y_d$;

From the first two rows of the table: MPC = (500-100) / (500-0) = .8

Part B:

3. If the only type of demand in this country's economy came from consumers (as shown in the C1 column) so that aggregate demand would equal C1, what would be the equilibrium level of GDP?

In equilibrium, expenditure (or, aggregate demand) equals output (E = Y). In our case the only type of expenditure is consumption (i.e., E = C), therefore, Y = C in equilibrium, and from the table we find that it happens when Y = 500.

Analytically: C = a + b*Y

(Here we do not distinguish between Y_d and Y).

Y = C → Y = a + b*Y → Y = a / (1 - b) → Y = 100 / (1 - .8) = 500

(because a = 100 ← it is consumption when Y = 0).

4. Note the relationship between GDP and investment demand as indicated by the data in columns 1 and 4. What kind of cause-and-effect is being indicated?

Investment is a constant – there is no relationship between I_1 and GDP (investment is independent of GDP).

5. If C1 and I1 represent the current demand patterns of the household and business sectors of the economy, calculate the aggregate demand (Y2) at the various levels of GDP (Column 5). What will the equilibrium level of GDP be in this case? Note the level of consumer spending and investment spending once this new macroeconomic equilibrium is reached. What has been the effect of this increase in investment demand?

E = C + I,

and in equilibrium

E = Y →

Y = a + b*Y + I → Y = (a + I) / (1 – b).

Y = (100 + 200) / (1 - .8) = 1,500

As we see, increase in investment demand by 200 (from 0 to 200) leads to 1,000-unit (from 500 to 1,500) increase in equilibrium GDP. This is consistent with the value of the multiplier: 1 / (1 – b) = 1 / (1 - .8) = 5

6. Now let investment demand fall to I2 because of higher interest rates. Calculate the new aggregate demand (Y3) pattern and the new GDP equilibrium. Note the levels of consumption and investment spending at this next equilibrium. Given the change in investment demand that brought about this new equilibrium, how big is the multiplier? (Check this out by using the multiplier formula provided in class and the data for Y3.) Why is there a multiplier effect working in this economy?

Again,

Y = (a + I) / (1 – b),

where now I = I_2.

Y = (100 + 100) / (1 - .8) = 1,000

ΔY = multiplier * ΔI → 500 = 5 * 100

Multiplier = 5 (see the previous question).

Multiplier works because money spent by one party is money received by another party and it goes to the next round of spending. An increase in equilibrium expenditure increase consists of two parts: autonomous increase (ΔI in our case) and induced increase because people who are paid that investment money now have more money to spend.

7. Next, switch to the third investment demand pattern, I3 (column 8). This column is assumed to be related to the first (GDP) column. What cause-and-effect is being implied in the data in columns 1 and 8? Does this make sense? Calculate Y4 (column 9) from C1 and I3 and find the new equilibrium. Note the levels of consumption and investment spending at this equilibrium.

Now the investment increases linearly as GDP rises. This makes sense: bigger GDP means greater economic activity, and this makes businesses think about bigger investments.

From the table we see that in equilibrium now Y = 1,500.

Calculations:

E = C + I;

C = a + b*Y;

I = d + e*Y → e = ΔI / ΔY = (100 - 50) / (500 - 0) = .1; d = I(when Y=0) = 50.

Equilibrium: Y = a + b*Y + d + e*Y → Y = (a +d) / (1 - b - e).

Y = (100 + 50) / (1 - .8 - .1) = 1,500.

C = 100+.8*1,500 = 1,300.

I = 50+.1*1,500 = 200.

8. Now, assume that technological breakthroughs occur (e.g., some sort of feasible superconductivity) which boost investment demand to I4 (column 10). Calculate the new Y5 and the new GDP equilibrium. What multiplier seems to be working in this case? Does this match up with the slope of the aggregate demand line indicated by the Y5 data? Note the new amounts of consumption and investment spending that occurs with this Y5 equilibrium and compare them with the corresponding amounts with Y4.

We use the formulas from the above question, just use I4 instead of I3 now.

I = d + e*Y → e = ΔI / ΔY = (200 - 150) / (500 - 0) = .1; d = I(when Y=0) = 150.

Y = (a +d) / (1 - b - e) = (100 + 150) / (1 - .8 - .1) = 2,500.

The reason for the boost is an autonomous rise in investment -- Δd = 100.

Multiplier = 1 / (1 - b - e) = 10.

This is the slope of the aggregate demand line: ΔY5 / ΔGDP = (700-250)/(500-0) = 0.9.

C = 100+.8*2,500 = 2,100. ΔC = 2,100-1,300 = 800 = b*ΔY

I = 150+.1*2,500 = 400. ΔI = 400-200 = 200 = Δd + e*ΔY.

Extra! Extra! Read All About It! Military Coup Overthrows Philosopher King!

Yes, the generals are now in control and the Kingdom of Philhill has been replaced by the Republic of Hawk Heaven. And, of course, a lot of military government spending will be necessary to ensure domestic tranquility (?) and deter foreign invasions. The public budget will rely on **income taxation** for financial support. The following macroeconomic picture results from these changes:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
At a GDP of :	T1	Yd1	C1	I1	G1	C1+I1+G1= Y1	G2	C1+I1+G2= Y2	T3	Yd3	C3	I3 (=I1)	G3 (=G1)	C3+I3+G3= Y3
0	0	0	100	100	1,000	**1,200**	**1,240**	**1440**	**-200**	**200**	**280**	100	1,000	**1,380**

600	200	400	460	100	1,000	**1,560**	**1,240**	**1800**	**0**	**600**	**640**	100	1,000	**1,740**
1,200	400	800	820	100	1,000	**1,920**	**1,240**	**2160**	**200**	**1000**	**1000**	100	1,000	**2,100**
1,800	600	1,200	1,180	100	1,000	**2,280**	**1,240**	**2520**	**400**	**1400**	**1360**	100	1,000	**2,460**
2,400	800	1,600	1,540	100	1,000	**2,640**	**1,240**	**2880**	**600**	**1800**	**1720**	100	1,000	**2,820**
3,000	1,000	2,000	1,900	100	1,000	**3,000**	**1,240**	**3240**	**800**	**2200**	**2080**	100	1,000	**3,180**
3,450	**1150**						**1,240**	**1240**	**950**	**2500**	**2350**	100	1,000	**3,450**
3,600	1,200	2,400	2,260	100	1,000	**3,360**	**1,240**	**3600**	**1000**	**2600**	**2440**	100	1,000	**3,540**
4,200	1,400	2,800	2,620	100	1,000	**3,720**	**1,240**	**3960**	**1200**	**3000**	**2800**	100	1,000	**3,900**

Part C:

9. Note the portion of GDP that the T1 tax system absorbs. What cause-and-effect relationship is being described in the data of columns 1 and 2?

In the Republic, tax is a linear function of GDP. The bigger is the Republic's output, the more taxes are collected.

10. Note that the households have changed the amount of consumption demand (Eat, drink, and be merry for tomorrow ... ?) they show at each level of GDP (compared to the situation in Parts A and B). Calculate the country's MPC now.

$C = a + b*Y_d$ → MPC = b = $\Delta C / \Delta Y_d$ = (460-100) / (400-0) = .9.

Because of a political instability, MPC is bigger now (i.e., on average, people spend more cents out of every dollar earned than before).

11. Note the data in columns 1 and 6 (G1). What cause-and-effect is being assumed between government spending and GDP? Does this sound plausible to you? (Why?/Why Not?)

Government spending is assumed to be independent of GDP -- 1000 units for any GDP level. This is implausible assumption - G should be financed (usually by tax revenue).

12. Use aggregate demand Y1 (= C1+I1+G1) to determine the republic's initial equilibrium level of GDP. Indicate what the levels of consumer spending, investment spending, government spending, and the government's budget deficit (G1 - T1) will be at this equilibrium.

E = C + I + G;

$C = a + b*Y_d$; → a = C(when Y_d=0) = 100.

$Y_d = Y - T$;

T = Tbar + t*Y; → t = $\Delta T/\Delta Y$ = 200/600 = 1/3; Tbar = T(when Y=0) = 0.

t is an income tax rate, and in our case it is 33.33%

In equilibrium E = Y → Y = a + b * (Y - Tbar - t * Y) + I + G;

Y = [1 / (1 - b + b * t)] * [a - b * Tbar + I + G];

$Y = [1/(1-.9+.9/3)] \times [100-0+100+1000] = 3000$

Part D:

13. Calculate the slope of the aggregate demand line (C1 + I1 + G1) and the size of the multiplier for this economy.

Slope of the aggregate demand line is: $b*(1-t)=0.6$
Multiplier is: $1/(1-b+b*t)=2.5$

14. Suppose that defense spending increases by 240 because of an increased threat of invasion. Show this change as G2 in the table above and calculate the new equilibrium level of GDP (Y2). At the new equilibrium what are the values of consumption spending, investment spending, taxes, government spending and the government's budget deficit. Does this result match up with the multiplier you just calculated?

ΔY = [1 / (1 - b + b * t)] * Δ G;

ΔY = 2.5 * 240 = 600,

so Y = 3,600.

T = (1/3)*3,600 = 1,200.

C = 100 + .9 * (3,600 - 1,200) = 2,260.

I = 100.

G = 1,240.

Government deficit = G - T = 1,240 - 1,200 = 40.

15. The government's economists have estimated that this economy would be at "full employment (or natural level)" if GDP was 3,450. Return to the initial equilibrium of question 12, based on the original G1. Given the multiplier that you estimated above (question 13), how big an *autonomous* change in aggregate demand would be needed to move this economy from the question 13 equilibrium to an equilibrium at 3,450?

Autonomous change in aggregate demand is a parallel shift of AD curve. In our case it is Δa in consumption function (we could equally assume ΔI). Using the basic formula from the question 12, we can derive:

ΔY = multiplier * Δa → Δa = ΔY / multiplier = (3,450-3,000)/2.5 = 180.

16. If a tax change is used as the means of moving the economy to full employment, calculate how large this tax change should be (Assume that the *slope* of the tax line vis-a-vis GDP, representing how tax revenue responds to changes in GDP, does not change; i.e., all tax changes will be changes in *autonomous (poll or head) taxes*.) Show this new tax amount as T3 in the table above, with its accompanying impact on other variables up to Y3. How large will C, I, G, and the government deficit be at the new 3,450 equilibrium?

Changes in autonomous taxes mean change in Tbar, i.e., parallel shift of the tax schedule.

Using the formula from the question 12,

ΔY = [-b / (1 - b + b * t)] * ΔTbar → ΔTbar = ΔY / [-b / (1 - b + b * t)].

ΔTbar = 450 / (-.9 * 2.5) = -200.

Since Tbar was 0, so now it is -200.

To get T3, we subtract 200 from T1 column.

T = -200 + (1/3)*3,450 = 950.

Y_d = 3,450 - 950 = 2,500.

C = 100 + .9*3,450 = 2,350.

I = 100 and G = 1,000 -- no change.

Government deficit = G - T = 1,000 - 950 = 50.

17. Suppose that a change in government spending is used *instead* of the above tax change to achieve full employment. How big must the government spending change be? How large will C, I, G, and the government's budget deficit be at the 3,450 equilibrium, when adjustment is made in this fashion?

ΔY = multiplier * ΔG → $\Delta G = \Delta Y$ / multiplier.

ΔG = 450 / 2.5 = 180.

C = 100 + .9 * (3,450 - 1,150) = 2,170,

where 1,150 = 3,450*(1/3) - income tax on the old schedule (T1 when Y = 3,450).

I = 100 and G = 1,000 -- no change.

Government deficit $= G - T = 1180 - 1150 = 30$ -- this is a deficit